

Problem of the Month Diminishing Return

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: “Make sense of problems and persevere in solving them.” The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:

In the Problem of the Month *Diminishing Return*, students use number operations, organized lists, and probability to solve problems. The mathematical topics that underlie this POM are knowledge of number sense, comparison, subtraction, division, rates, rational numbers, and conditional probability. The mathematics in this POM includes converting repeating decimals to fractions and using algebraic reasoning.

In the first level of the POM, students are presented with a situation that asks the students to determine the number of helpers needed to carry drinks. Each helper carries a uniform amount. The students determine the number of helpers that are needed to do the job and compare how two classes complete the job with different numbers of helpers. Each class needs helpers to carry drinks, but the number of drinks the two sets of helpers can carry differs. So students determine which class needs more helpers. In level B, students have a given amount of money and a list of possible toys to purchase. Their goal is to determine a list of toys whose prices sum to the exact amount provided. In level C, students are asked to consider a problem that involves rate. Two people can mow the lawn at differing rates. The challenge is to determine a time when the lawn left to mow is twice the size of the other person's lawn. In level D, the student determines how much weight a person has lost if each month that person loses weight at a diminishing level. In level E, students are asked to find the exact theoretical chance knowing three conditional probable events expressed as repeating decimals.

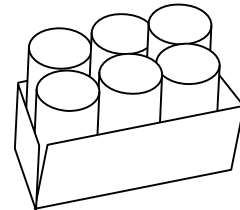
Problem of the Month

Diminishing Return

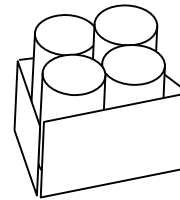
Level A:

Some classes are going out for a picnic lunch. The teachers bought drinks in packs for their classes.

Thirty-three students are in Mrs. Browne's class. Mrs. Browne bought six-packs for her class. She needs helpers, so she picks students to carry one six-pack each.








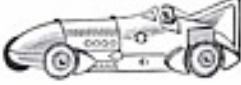













Twenty-two students are in Mrs. Robinson's class. Mrs. Robinson bought four-packs for her class. She needs helpers, so she picks students to carry one four-pack each.



Which teacher had to pick more helpers?

Show how you found your answer.

Level B:

Amusement Center Store					
					
Yo Yo \$1.22	Doll \$2.75	Duckie \$1.85	Tractor \$5.97	Airplane \$6.47	
					
Ball \$2.16	Racecar \$7.13	Dog \$4.57	Jump Rope \$1.46	Car \$5.18	
					
Elephant \$3.16	Bear \$4.89	Xylophone \$7.11	Tank \$6.45	Checkers \$4.77	
					
Boat \$8.04	Train \$6.71	Jacks \$2.31	Truck \$6.21	Whistle 98¢	Pinwheel 87¢

Mia has earned \$43.94 of tokens playing games at the amusement center. The store in the amusement center has the following toys for sale. She plans to get toys and donate them to a local charity for needy children. The tokens are only good in this store, so she plans to spend all the tokens. What combinations of toys can she buy in order to spend all the tokens?

Show how you found your solution.

Is your solution the only possible answer? Explain.

Level C:

Maxine and Sammie have the same size lawn. Maxine can mow the lawn in 24 minutes and Sammie can mow the lawn in 36 minutes. At what time will Sammie have twice as much lawn to mow as Maxine?

Maxine and Sammie have to also mow their parking strips that are the same size. Maxine can mow the parking strip in 6 minutes and Sammie can mow the parking strip in 9 minutes. At what time will Sammie have twice as much grass to mow as Maxine?

Level D:

Rollie was successful in losing weight. He had a goal weight in mind. He went on a diet for three months. Each month, he would lose one-third of the difference between his current weight and his goal weight and an additional three pounds. At the end of three months, he was just 3 pounds over his goal weight. How many pounds did he lose in those three months?

Explain how you arrived at your solution.

Level E:

The probability of being born a male is $0.\overline{466}$. The probability of being born in North America is $0.\overline{153846}$. The probability of being born in an urban location is $0.\overline{3571428}$. Find the exact probability that a baby will be born a male, in North America, in an urban location.

Explain the method you used to find your solution.

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Primary Version Level A

Materials: A picture of the four-pack and six-pack of drinks

Discussion on the rug: (Teacher holds up the pictures of the four pack) "Here is a picture of a drink holder. How many drinks does this holder have?" (Teacher holds up the pictures of the six-pack) "Here is a picture of a different drink holder. How many drinks does this holder have?" (Teacher asks students to explain their answers. Some students come forward and demonstrate to the class how they arrived at their answer.) (Showing the four-pack) "If we have two helpers and each helper carries one of these holders of drinks, how many drinks will we have in all?"

In small groups: (Each student has access to counting manipulatives. Teacher asks the following questions. Only go on to the next question if students have success.)

If 12 of us are going on a picnic, how many drink holders of this size (four-pack) do we need to take?

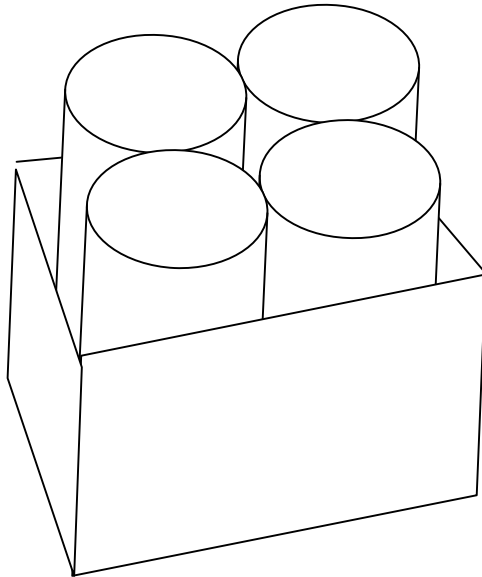
If 12 of us are going on a picnic, how many drink holders of this size (six pack) do we need to take?

Why do we need fewer six-packs than four-packs?

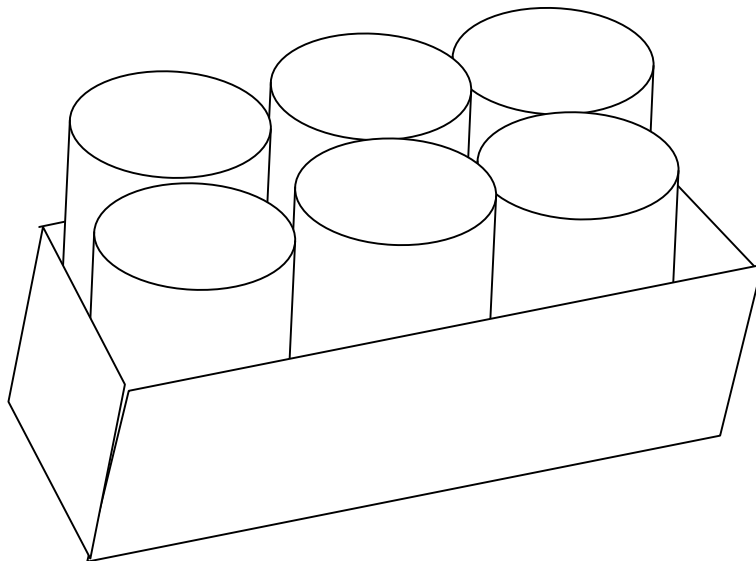
(If students are successful with these values, increase the size of the students going on a picnic, careful to ask for multiples of four and three (12, 24, 36, etc.)

At the end of the investigation, have students either discuss or dictate a response to this summary question.)

Tell me how you figured out your answer and how do you know.



4 - Pack



6 - Pack

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Task Description – Level A
This task challenges a student to find the number of equal groups and interpret any remainder. Students may solve the task using repeated addition or subtraction. Students may also solve the problem using division.
Common Core State Standards Math - Content Standards
<p><u>Operations and Algebraic Thinking</u></p> <p>Represent and solve problems involving addition and subtraction. 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, putting together, taking apart, and comparing with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the positions.</p> <p>Represent and solve problems involving multiplication and division. 3.OA.2 Interpret whole-number quotients of whole numbers, e.g. interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 6 shares, or as a number of shares when 56 objects are portioned into equal shares of 8 objects each.</p> <p>3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p>Solve problems involving the four operations, and identify and explain patterns in arithmetic. 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p> <p><u>Number and Operations in Base Ten</u></p> <p>Use place value understanding and properties of operations to perform multi-digit arithmetic. 4.NBT.6 Find whole-number quotients and remainders with up to four digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or models.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to <i>decontextualize</i> – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to <i>contextualize</i>, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</p> <p>MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student</p>

might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Task Description – Level B
This task challenges a student to analyze costs for several items to find a specific total. Students are then challenged to find alternate solutions. Students may use an organized list or chart with differences to help find substitutions, so looking at structure to see differences in costs can simplify the process.
Common Core State Standards Math - Content Standards
<p><u>Numbers and Operations in Base Ten</u> Perform operations with multi-digit whole numbers and with decimals to hundredths. 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p> <p><u>Measurement and Data</u> Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time liquid volumes, masses of objects and money, including problems involving simple fractions or decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit.</p> <p><u>Expressions and Equations</u> Reason about and solve one-variable equations and inequalities. 6.EE.6 Understand solving an equation or inequality as a process of answering a question: which values from a specified set if any, make the equation or inequality true? Use substitution to determine whether any given number in a specified set makes an equation or inequality true.</p> <p>Solve real-life and mathematical problems using numerical and algebraic expressions and equations. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals); using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p>

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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Task Description – Level C
This task challenges a student’s knowledge of rates and proportional reasoning to solve problems about mowing a lawn. Students can use many strategies, such as using the formula rate x time = amount mowed and using a table. The students can also use knowledge of equations to solve the problem.
Common Core State Standards Math - Content Standards
<p><u>Ratios and Proportional Relationships</u></p> <p>Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <ol style="list-style-type: none"> Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. Solve unit rate problems including those involving unit pricing and constant speed. <p><u>Expressions and Equations</u></p> <p>Reason about and solve one-variable equations and inequalities.</p> <p>6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p> <p>6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.</p> <p>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p> <p>7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.2 Reason abstractly and quantitatively.</p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to <i>decontextualize</i> – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to <i>contextualize</i>, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</p> <p>MP.4 Model with mathematics.</p> <p>Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical</p>

situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month
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Task Description – Level D
This task challenges a student to consider a recursive rule about weight loss over a 3-month period and find the starting weight. Students may approach the problem using guess-and-check or by writing equations to define the change in situation of the time of the diet.
Common Core State Standards Math - Content Standards
<p><u>Number and Operations - Fractions</u></p> <p>Use equivalent fractions as a strategy to add and subtract fractions. 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g. by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.</p> <p>Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p><u>Expressions and Equations</u></p> <p>Reason about and solve one-variable equations and inequalities. 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p> <p>6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.</p> <p>Solve real-life and mathematical problems using numerical and algebraic expressions and equations. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p><u>The Number System</u></p> <p>Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers. 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers, represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>7.NS.2 Apply and extend previous understandings of multiplication and division of fractions to multiplication and division of fractions to multiply and divide rational numbers.</p> <p>7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.</p> <p><u>High School – Algebra – Creating Equations</u></p> <p>Create equations that describe numbers or relationships. A-CED.1 Create equations and inequalities in one variable and use them to solve problems, include equations arising from linear and quadratic functions, and simple rational and exponential functions.</p> <p><u>High School – Algebra –Reasoning with Equations and Inequalities</u></p>

Solve equations and inequalities in one variable.

A-REI.3 Solve linear equations and inequalities in one variable, including equations and coefficients represented by letters.

A-REI.4 Solve quadratic equations in one variable.

Common Core State Standards Math – Standards of Mathematical Practice**MP.2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Task Description – Level E
This task challenges a student to work with repeating decimals and probabilities to calculate the probability of being born a male in an urban North American location.
Common Core State Standards Math - Content Standards
<p><u>The Number System</u> Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division of fractions to multiplication and division and of fractions to multiply and divide rational numbers.</p> <p>7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.</p> <p><u>Statistics and Probability</u> Investigate chance processes and develop, use and evaluate probability models. 7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies: if the agreement is not good, explain possible sources of the discrepancy.</p> <p>b. Develop a probability model (which may not be uniform by observing frequencies in data generated from a chance process.</p> <p><u>High School – Statistics & Probability – Conditional Probability and the Rules of Probability</u> Understand independence and conditional probability and use them to interpret data. SP-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or”, “and”, “not”).</p> <p>SP-CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p> <p>SP-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.</p> <p>Use the rules of probability to compute probabilities of compound events in a uniform probability model. SP-CP.8 Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$, and interpret the answer in terms of the model.</p>
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MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

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Task Description – Primary Level
This task challenges a student to think about equal size groups of objects arranged in an array. Students use counting, drawing, repeated addition or multiplication to solve simple problems.
Common Core State Standards Math - Content Standards
<p><u>Counting and Cardinality</u> Count and tell the number of objects. K.CC.5 count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configurations; given a number from 1-20, count out that many objects.</p> <p><u>Operations and Algebraic Thinking</u> Work with equal groups of objects to gain foundations for multiplication. 2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns.</p> <p>Represent and solve problems involving multiplication and division. 3.OA.1 Interpret products of whole numbers e.g. interpret 5×7 as the total number of objects in 5 groups of 7 objects each.</p> <p>3.OA.2 Interpret whole-number quotients of whole numbers, e.g. interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are portioned into equal shares of 8 objects each.</p> <p>3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to <i>decontextualize</i> – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to <i>contextualize</i>, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</p> <p>MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions.</p>

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